Multiple polynomial-based generation of DEM from topographic contour data

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Abstract:

The purpose of this paper is to introduce a methodology for DEM generation from 1/100,000 topographiccontoursbased on multiple polynomial interpolations. Sinai Peninsula, in Egypt, was chosen as a study area for its size and topographic diversity. The selected test area stretches north south from gently-slope flat land in northern part to rugged mountainous region in Sothern part. The proposed modeling scheme comprises five steps. In the first step, contours for Sinai Peninsulawere manually digitized from 1:100,000 scale topographic maps. The obtained data is about 940,624 points. As well, SRTM DEMs were downloaded and processed to be used for comparison. In the second step, theSRTM DEMs were classified into three classes: flat; rolling; and mountainous depicts slopes of less than 5%, from 5 to12% and greater than 12% respectively. In the third step, three DEMs with a 30m horizontal resolution were derived from the digitized contours using linear; quadratic; and cubic polynomial equations in a least-square fashion.In the fourth step, flat, rolling and mountainous areas were extracted from linear, quadratic and cubic-based DEMs respectively.A refined DEM was constructed by combining the three extracted areas in one scene. Finally, the reconstructed DEMwere compared withvariety of existing interpolation techniques. The comparison is based on four criteria: RMSE; systematic errors; derived slope; and computational cost. Statistics show that for the reconstructed DEM, the RMSE was 2.38m with no systematic errors. The derived slope and computational cost were comparable to the well known interpolation technique, Kriging, with simpler implementation.

Keywords: Polynomial**,** interpolation, DEM, elevation, slope.

1. Introduction

One of the most important data inputs for GIS applications and natural hazards mapping is an accurate DEM. DEM data provide risk managers and GIS practitioners with information about the slope and aspect of the terrain and are used heavily in models for landslides, floods, soil erosion, and watershed catchment area studies. Because of the many sources of elevation data available for GIS modeling, care must be taken in selecting the most appropriate data set for the desired application [Chirico, 2004].

DEM is a computer representation of the earth's surface from which topographic parameters such as slope, upslope area, and the topographic index can be digitally generated. Elevation data can be represented digitally in many ways, including a gridded model where elevation is estimated for each cell in a regular grid, a triangular irregular network, and contours. Representation of the DEM as a grid is quite common, as this format lends itself well to computer computations [NDEP, 2004].

DEMs are produced from a variety of source data but are most commonly produced fromcontours represented on topographic maps, derived from ground surveys, or triangulated fromstereo aerial photographs. Stereopairssuch as with SPOT, IRS, and ASTER satellite imagery is also an effective means ofproducing DEM data. RADAR technology, InSAR or IfSAR, is also a reliable means from which elevation data isobtained. Light detection and ranging (LIDAR) is rapidlybecoming the standard for developing high horizontal and vertical resolution DEMs.

Spatial Interpolation is the process of using points with known values to estimate unknown values at other points. Spatial interpolation methods can be categorized in several ways. They can be grouped into global and local methods. A global interpolation method uses every available known point to estimate an unknown value. On the other hand, a local interpolation method uses a sample of known points to estimate an unknown value. Also, spatial methods can be grouped into exact and inexact interpolation. Exact interpolation predicts a value at the point location that is the same as its known value. In contrast, inexact interpolation, predicts a value at the point location that differs from its known value. Finally, the spatial interpolation methods can be deterministic or stochastic. A deterministic interpolation method provides no assessment of errors with predicted value. On the other hand, a stochastic interpolation method, offers assessment of prediction errors with estimated variances [Mitas and Mitasova, 1999].

Abundant literature exists on methods for interpolation of DEM. Commonly used interpolation methods include the IDW (Inverse Distance Weighting) and various modifications of kriging. Kriging is a geostatistical method for spatial interpolation that proves usefulness and Popularity in many fields. Kriging depends on spatial and statistical relationships to calculate the surface. It differs from the other interpolation methods as it can assess the quality of the prediction with estimated prediction errors [Press et al., 2007].

Carlos [2006] found that variograms calculated with linear trend residues of topographic data have adequate fits to classical variogram models, this model can then be used by kriging to interpolate the whole dataset at a finer resolution which produced good results. Grohmannand Steiner[2008]studied how the proper adjustment of variogram and kriging parameters, namely the nugget effect and the maximum distance within which values are used in interpolation, can be set to achieve quality results on data resampling.

Her and Heatwole[2008] evaluated the potential and performance of seven interpolation techniques as a means of resolution refining. Inverse Distance Weighting (IDW) with exponents of 0.5, 1, and 2, regularized and tension Spline, Kriging, and Natural Neighbor interpolation were tested using

ArcGIS 9.1 Spatial Analyst. The obtained elevation, slope, aspect, sink, and stream networks from the interpolated DEMwere compared quantitatively with those derived from the original DEM. The results showed that the interpolation techniques were not significantly different in deriving elevation, total flow length, and delineating watersheds. However, all methods produced shallower slopes than the original DEM and had a smoothed surface with fewer sinks.

The suitability of various interpolation methods for creating elevation models has been studied in several papers. Chaplot et al. [2006] concluded that the performance of the methods varied significantly depending on the density of the source data and the flatness of the modeled earth surface. No single method was optimal in all conditions. The most important factor for selecting a DEM interpolation method is the interpolation function that can be adapted to the varying terrain character. Other criteria that may influence the selection of a particular method are the degree of accuracy desired and the computational effort involved [Pohjola et al., 2009].

This paper presents a methodology that DEM users can apply for DEM generation from topographic contours based on multiple polynomials interpolation. The multiple polynomials interpolation can assist in deriving accurate DEMs for landscapes with varying terrain character. The rest of the paper is organized as follows. A description of the study areais given in the first section of the paper.The second section describes the proposed methodology. The third section presents the experimental results of our algorithm and finally the fourth section deals with the conclusion and possible extension for the future work.

2. Study Area

Sinai Peninsula is located in the northeastern of Egypt. It extends between32º E and 35º E, and between 27.5º N and 31.5º N. Sinai Peninsula was chosen as a study area for its size and topographic diversity, which includes not only narrow ravines and steep ridges, but also exhibits coastal plain topography along its edges as shown in Figure 1. The Peninsula contains several mountain ranges with summits reaching up to 2,647m above sea level.

Figure 1: Landsat 7 ETM+Satellite Image showing general topographic relief of Sinai Peninsula.

3. Methodology

The flow chart for the methodology of this study is represented in Figure 2.

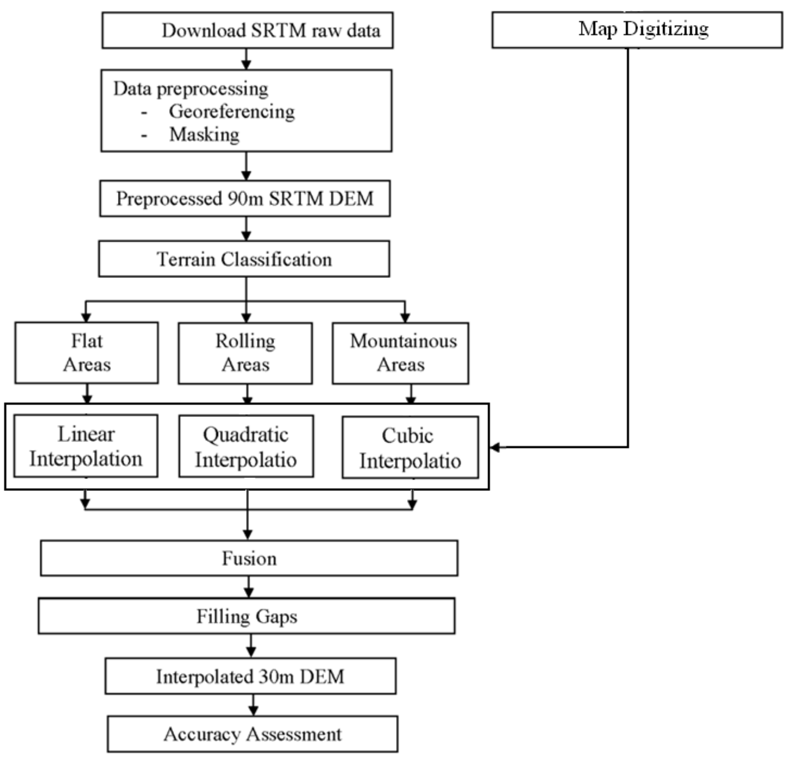
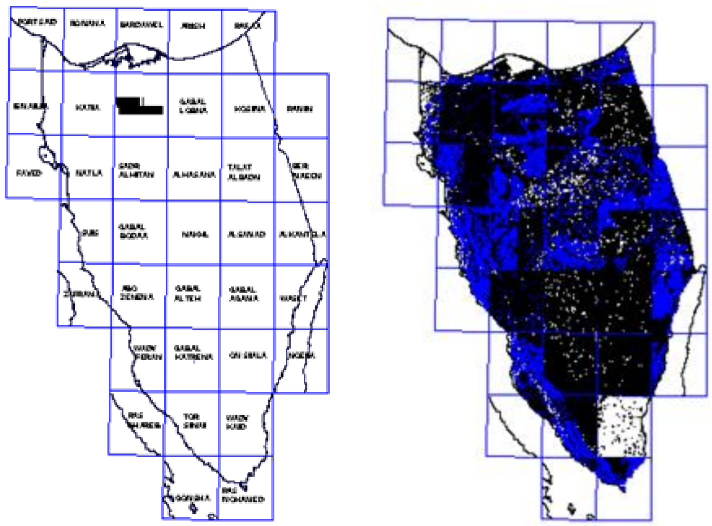


Figure 2: The flow chart for the proposed methodology.

3.1.Contour Derived DEM

Contours for Sinai Peninsulawere manually digitized from 1:100,000 scale topographic maps. Sinai Peninsula is covered by 36 maps. The maps were produced from aerial photographs taken in 1951 and updated with additional aerial photography in 1978. The topographic maps have a contour interval of 20m. Based on National Map Accuracy Standards (NMAS) that 90% of true elevation values are within one-half of the contour interval, the vertical accuracy for a DEM developed from these contours is estimated to be ±10m. AutoCad file format is obtained using the tablet digitizer with AutoCad driver. The obtained data is about 940,624 points (34,263 from spots and 906,378 from contours) as shown in figure 3. The elevations varied from zero to 2639m with mean elevation 435m and RMSE ±3.65m. The average intensity is 15.54pt/km^2. The AutoCad file (contours and spots) was exported as data file format in order to be suitable for most DEM software.



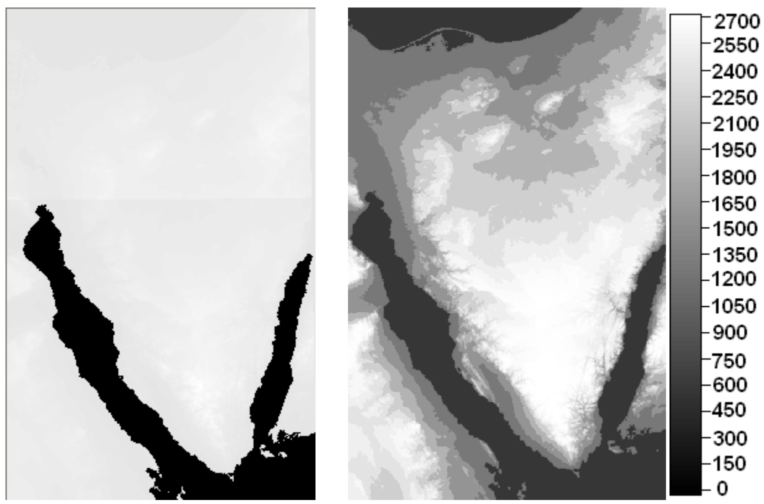
(a) *(b)*

Figure 3: Preparation of reference data: (a) Key maps, (b) digital maps.

3.2.Data Pre-processing

SRTM DEMs were freely downloaded from SDDS website at http://seamless.usgs.gov/. In order to cover the test area, two DEM files, half latitude each in size, were downloaded containing two files of 90m SRTM DEMs. After obtaining SRTM DEMs from SDDS website, all data had to be preprocessed because there are some typical problems that could be found from the original SRTM DEMs. One of them is the existing of data voids. Voids are points on the SRTM data that contain error data, this happens due to bad radar scattering, which can be defined into 2 types: the first one contains zero value, which normally appear as large planes around mountainous areas. The other one in contrast, appears as speckle noises in flat land areas. For the pre-processing step, in the case of error with large area, all were made buffers and were not considered for further statistic calculations. While errors that appeared to be speckles were re-calculated with 3x3 averaging window. The process was done by averaging 8 surrounding pixel value and replaces the center value (the error) with the new one. This process kept the regularity of grid-like data for further processing.

Datum adjustment calculations were performed on the DEM data to adjust all source data to elevations based on the WGS84/EGM96 geoid. In order to ensure proper elevation values, water features were masked. This is because coastal areas are often dropped out by the SRTM data because RADAR is absorbed and not returned to the sensor. It is worth mentioning that, the effect of merging scenes averages elevation values recorded in coincident areas. As well, SRTM DEMs over Sinai are actually DEMs since no dense forests or vegetation cover exists in the study area. Figure 4 shows the SRTM data before and after pre-processing



(a) *(b)*

Figure 4:(a) downloaded SRTM data; (b) processed SRTM data.

3.3.Slope Classification

The slope measurement is selected to classify the terrain into three classes: flat; rolling; and mountainous depicts slopes of less than 5%, from 5 to12% and greater than 12% respectively. Figure 5 shows slope calculations for the test area from the SRTM data.

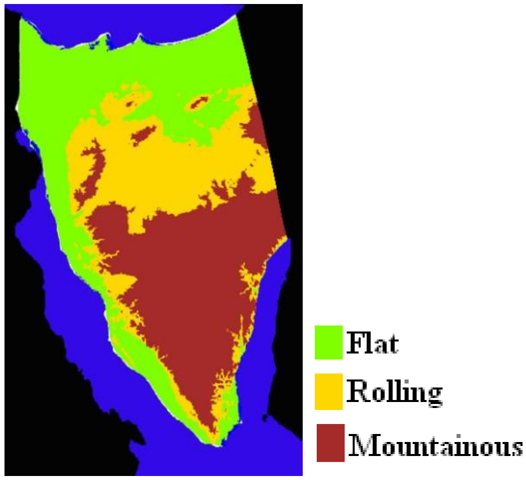


Figure 5: Slope calculations (in percent) from 90m SRTM DEM.

3.4.Multiple polynomial Interpolation

In the third step for multiple polynomial interpolations, three DEMs with a 30m horizontal resolution was created using the TOPOGRID command in ArcGIS. Each height value is computed by choosing 16 nearest sampling points to determine the parameters of three types of polynomials which are: Linear; quadratic; and cubic polynomial equations in a least-square fashion. The results are three raster grid arrays representing orthometric heights of the test area with 30m horizontal resolution as shown in figure 6.
For the first DEM, terrain was modeled with a first order polynomial as an inclined plane with the formula:

$$Z(x, y) = b_o + b_1 x + b_2 y + \varepsilon (1)$$

z: modeled z value.
x, y: the set of x and y coordinates of a particular point.
b_o, b_1, b_2: polynomial coefficients.
ε: the residual (the error in the interpolation process)

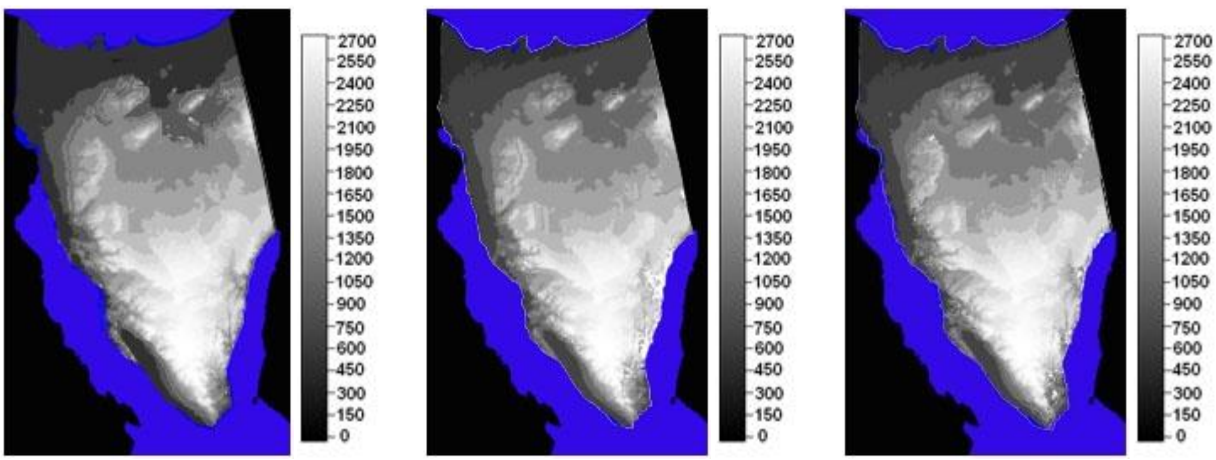
For the second DEM, a second order polynomial (quadratic), was applied in order to provide a better fit. A second order trend surface would be an undulating surface with the formula:

$$Z(x, y) = b_o + b_1 x + b_2 y + b_3 x^2 + b_4 xy + b_5 y^2 + \varepsilon (2)$$

For the third DEM, A third order polynomial, cubic, with a total of ten terms has been applied. It is more convoluted than the second order polynomial. It is worth mentioning that trend surfaces higher than third order not only include even higher powers of x and y but also more cross product terms, such as $b_4 x\, y$. On the other hand, problems with polynomials of high order is that the values in between the known points could be very wrongly estimated, as the surface forms *peaks* which can be much higher or lower than the range of the known data points. This is known as the *Runges phenomenon*.However, in order to use a lower order of polynomial for a more complex interpolation purpose, it is necessary to have excessive data and use the well-known calculation technique, least square adjustment.Bicubic polynomial is used basically to form 10 equations which are the relationships between height value (z) and horizontal parameters (x, y) in order to solve 10 unknown coefficient values by the process of least square. General form of bicubic polynomial is shown in equation (3).

$$z = a_1x^3 + a_2y^3 + a_3x^2y + a_4xy^2 + a_5x^2 + a_6y^2 + a_7xy + a_8x + a_9y + a_{10} \qquad (3)$$

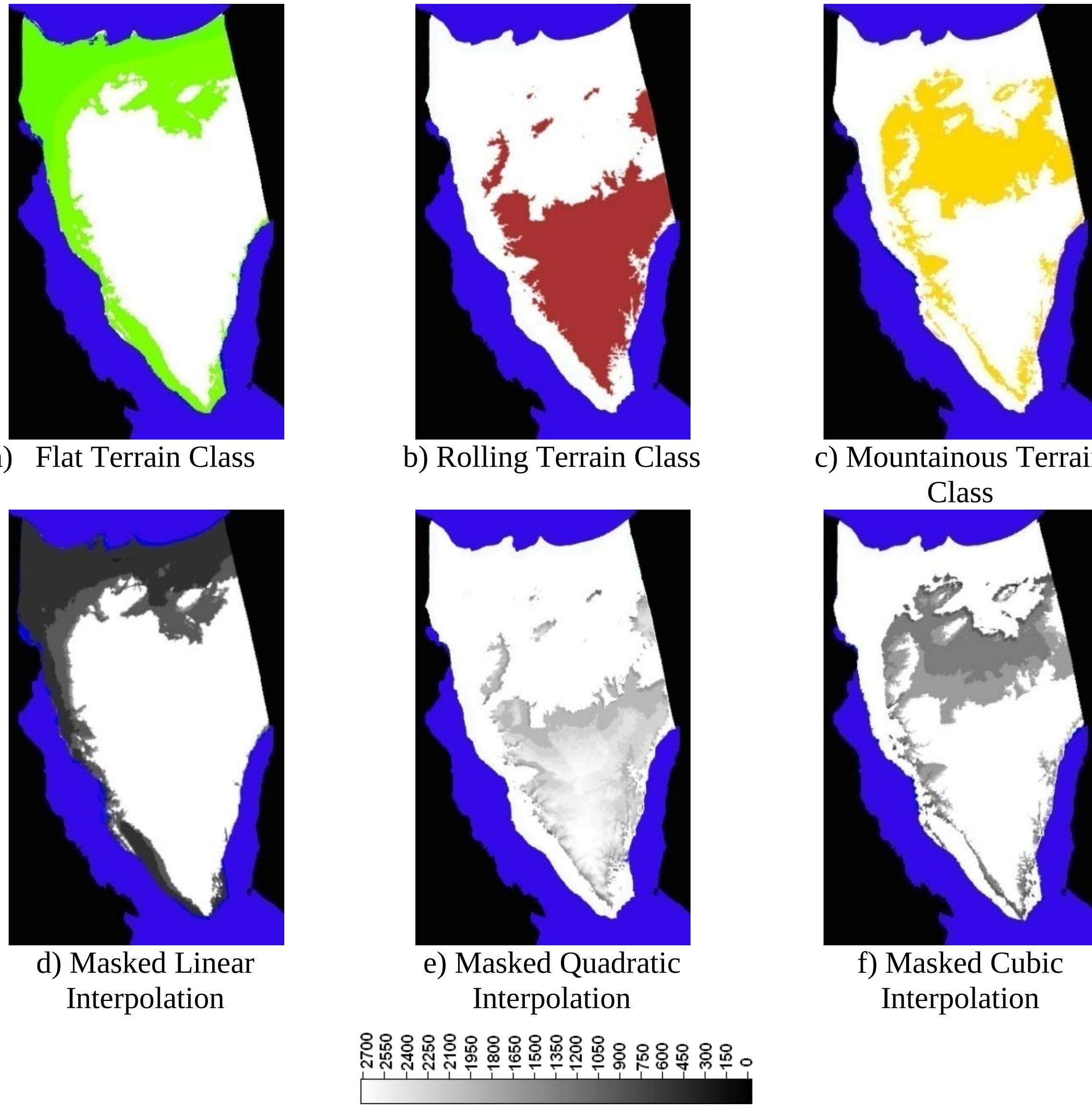
The results of the interpolation process are three DEMs for Sinai Peninsula, one for each interpolation technique, as shown in figure 6.



Linear interpolation results Quadratic Interpolation results Cubic interpolation results

Figure 6: Linear, quadratic and cubic interpolation results.

In order for each interpolation results to be effectivelyrepresent the corresponding terrain class, the classified image was then used to create three new images, one for each class, as shown in figure 6 (a, b and c). After that, each terrain class was compared with the corresponding interpolated DEM.In this respect, the pixels in the interpolatedDEM that belong to a given class remained the same, while the other pixels are masked out. The results of this comparison are three masked DEMs as shown in figure 7 (d, e and f).

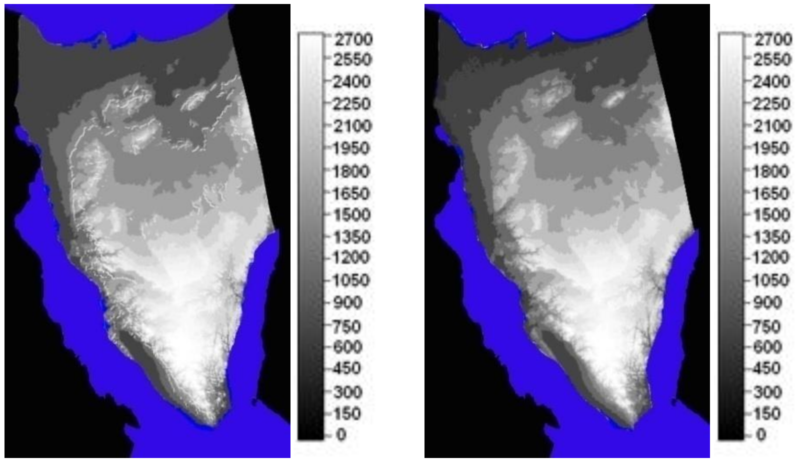


a) Flat Terrain Class
b) Rolling Terrain Class
c) Mountainous Terrain Class
d) Masked Linear Interpolation
e) Masked Quadratic Interpolation
f) Masked Cubic Interpolation

Figure 7: Generation of the masked DEMs.

3.5.Data fusion

In the forth step for data fusion, the refined DEM was constructed by combining the threemasked DEMs.As shown in figure 8(a), gaps can be observed at edges of classes due the fitting behavior of the polynomial interpolation. For pixels with no corresponding height value, an interpolation process from neighboring pixels was applied to avoid introducing new height values. The final refined DEM is shown in figure 8 (b).



(a) *(b)*

Figure 8:(a) data gaps in white around the three terrain classes, (b) final refined SRTM DEM after filling gaps.

3.6.Accuracy Assessment

In order to meet the objective, the 30m grids were created using a variety of point-based interpolations from the source data and compared with the proposed method using Golden Software Surfer9. These methods include: (1) Inverse Distance to a Power; (2) Kriging; (3) Minimum Curvature; (4) Natural Neighbor; (5) Nearest Neighbor; (6) Polynomial Regression; (7) Radial Basis Function; (8) Modified Shepard's Method; (9) Triangulation with Linear Interpolation; (10) Moving Average; (11) Data Metrics and (12) Local Polynomial.

Theoretical estimates of the generated DEM, against GPS surveyed heights have been performed. Only six points, evenly distributed over the test area, were available and have been applied for this comparison. The points were surveyed with Leica GPS-500 and GPS-1200 systems. Each point was measured in the static mode with over 30-minute observation sessions. The expected precision of GPS positioning is a few millimeters in horizontal coordinates and 1-2cm in vertical (height) coordinates.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(y_i - y_j)^2}{N-1}} \quad (4)$$

Where y_i is an elevation from the refined DEM, $y_j$is the corresponding GPS surveyed heights and N is the number of sample points. The RMSE statistic is essentially a standard deviation and is thus based on the assumption that errors in the DEM are random and normally distributed. The RMSE, while a valuable quality-control statistic, does not provide the DEM user with an accurate assessment of how well each cell in the DEM represents the true elevation. However, the true elevation can be expected to exist within the range provided by the RMSE.Under such an observation, three additional criteria (systematic errors; derived slope; and computational cost) were used for the comparison. In this regard, the SRTM DEM has been applied as a reference.

4. Results and Analysis

4.1.Model validation and testing

In order to evaluate the potential and performance of the proposed method, the refined DEM was compared with the GPS surveyed heights.The RMSE was between 0.82, 2.33 and 3.41m for flat, rolling and mountainous areas respectively. The magnitudes of maximum error were -4.74, 9.3 and 12.63m for flat, rolling and mountainous areas respectively. RMSE values for mountainous terrain is slightly higher than those for rolling and flat terrain. This DEMonstrates that in areas of higher elevation the magnitude of error increases. These results conform to the findings of Chirico [2004]. A possible reason for that can be that flat topography is more simply modeled while mountainous topography is more complex.Table 1presents the RMSE values and a statistical summarycalculated for the comparison. Using the interpolation methods, 12 different DEMs with a 30m resolution were created. Polynomial Regression provided the closest standard deviation to that of the proposed method. The values indicate that the RMSE for the proposed method is slightly better than the majority of other techniques.

Table 1: Statistical analysis of comparing interpolated elevations to corresponding reference elevations.

Method	Min	Max	Mean	RMSE
The Proposed Method	-4.٧٤	12.٦٣	9.٢٣	2.38
Inverse Distance to a Power	-5.٣٥	11.٣٨	9.٥٧	2.٤٦
Kriging	-3.٢٧	11.١٦	9.١٥	2.٦٢
Minimum Curvature	-4.١٤	11.٩٦	9.١٩	2.٠٧
Modified Shepard's Method	-4.٨٦	11.٢٧	9.٤٦	2.٧٠
Nearest Neighbor	-4.٢٨	11.٨٥	9.٣٥	2.٤٥
Polynomial Regression	-4.٢٥	10.٥٥	8.١١	2.٣٥
Radial Basis Function	-4.٣٨	11.٩٣	9.٥٤	2.١٤
Moving Average	-7.٧٣	12.٦٨	8.٣٢	3.٨٨
Data Metrics	31.٢٧	60.٧١	56.٢٠	5.٢٥
Local Polynomial	-4.١٦	10.٦٢	9.٠٨	2.٤٣

It is worth mentioning that while applying the *Natural Neighbor* and *Triangulation with Linear Interpolation* techniques, an internal application error has been detected by Golden Software program. This error reflects a bug or unexpected condition in the program and was reported to Golden Software technical support so it can be fixed.

4.2.Systematic errors

Regardless of data quality or production method, all DEMs are subject to systematic error. Systematic error is defined as being error occurring in some fixed pattern and introduced by data collection procedures and systems [NDEP, 2004]. The goal of this section is to check the presence of systematic errors with the proposed method through the comparison of the refinedDEM to the SRTM one.In Figure 9, positive values (areas where the refined values are higher than the reference ones) have green color, while negative values (areas where the refined values are lower than the reference ones) have red color. The random distribution of the differences indicates that the differences are not systematic, and hence no systematic biases have been occurred during the generation of the interpolated 30m DEM.
It is worth mentioning that, the majority of positive and negative differences are located around coastal areas. A possible reason for that could be the low intensity of data available at these areasas can be observed in figure 4(b). Another possible reason can be that the refinedDEM was generated from paper topographic maps. Some of these maps are outdated and may not incorporate topographic changes due to development occurrences around these coastal areas.

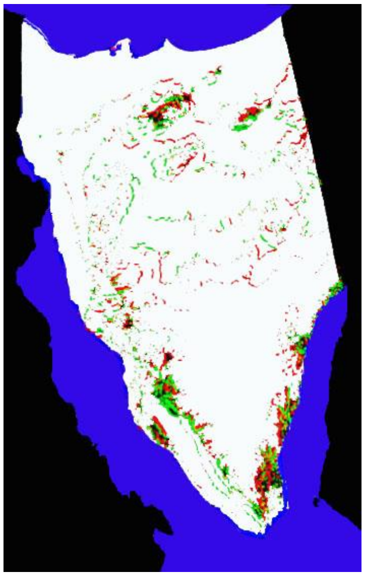


Figure 9: Scatter plot of positive and negative errors.

4.3.Derived Slope

The obtained slopes from the interpolated 30m DEMs were compared quantitatively with those derived from the SRTMDEM. Figure 10 showsthedirection and magnitude of the obtained slopes from each grid. The interpolation techniques were not significantly different in deriving slopes. However, *Polynomial Regression* and *Moving Average* interpolation techniques have clearly observed bias in the slope which was not found in the proposed method. The *Nearest Neighbor* interpolation showed gaps in slope, *Kriging* did not have significant weaknesses.

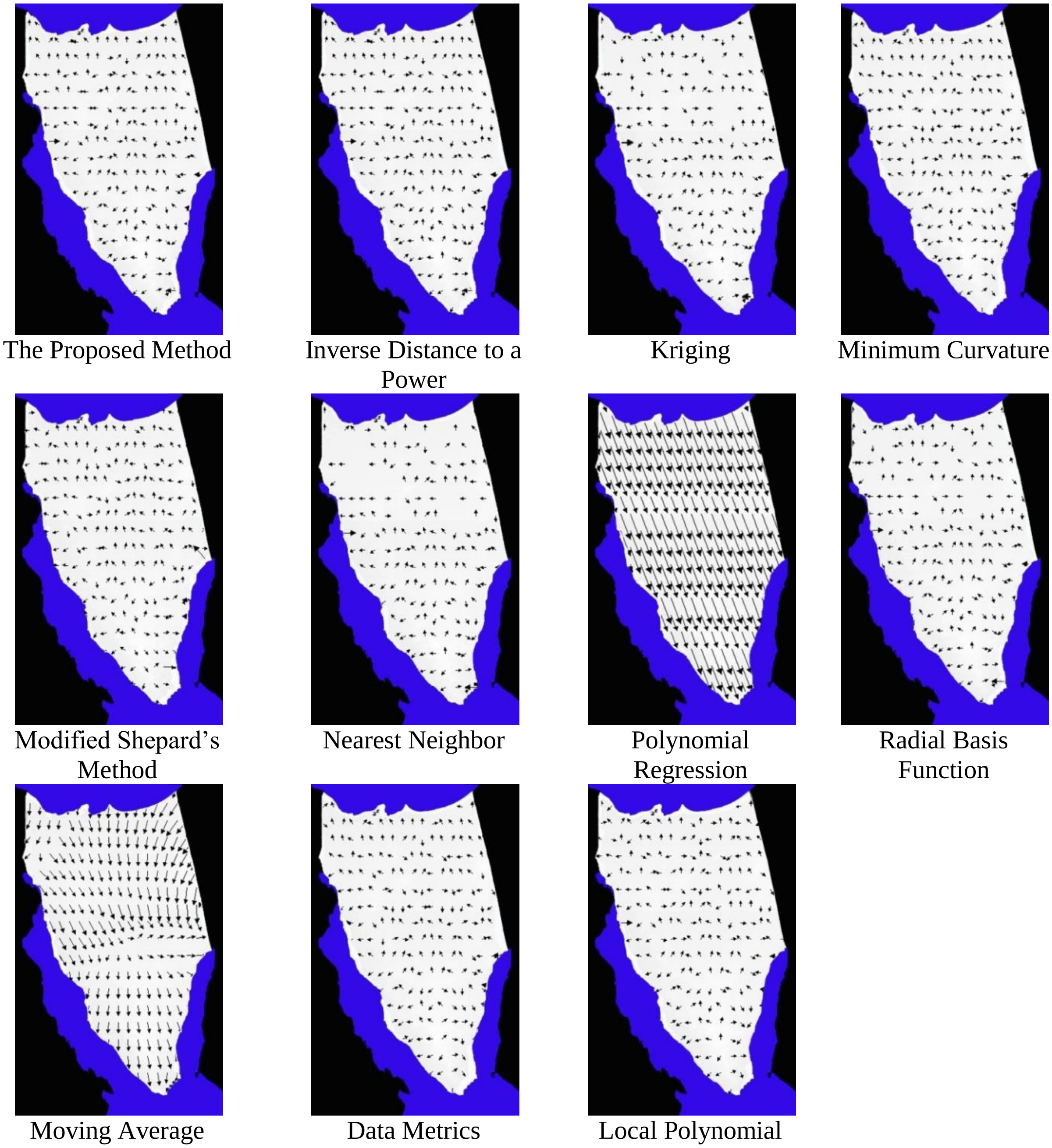


Figure 10: The obtained slopes for the proposed method as compared with existing techniques.

4.4.Computational Cost

The computational cost involved in implementing the proposed method is comparable to that of other interpolation methods as shown in Figure 11. It is worth mentioning that the processing time for the proposed method has been reduced by splitting the large test area into three smaller parts, processing each part separately and combining the results later.

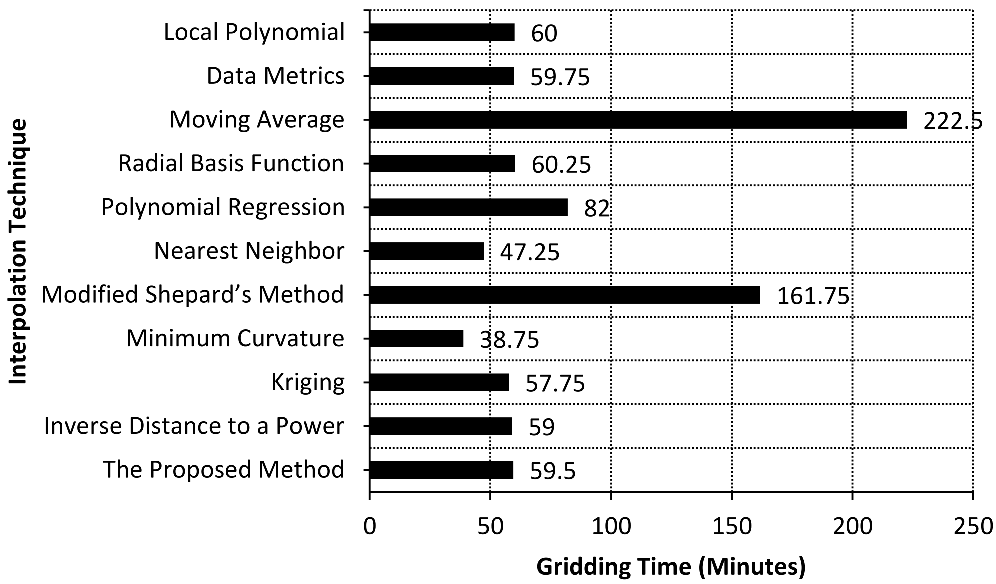


Figure 11: Computational cost comparison of the proposed method with existing algorithms.

5. Conclusions

In this paper an approach for contour-based DEM generationbased on multiple polynomials interpolation is described.The proposed method comprises five steps namely: data pre-processing; slope classification; multiple polynomial interpolations; data fusion; and validation. The results show that a DEM of coherent angular properties between neighbouring pixels was reconstructed successfully. The method showed results comparable to kriging interpolation with less complexity in the whole working process. The vertical accuracy of the interpolatedDEM was assessed with accurate GPS surveyed heights. The RMSE was found to be 2.38m. This figureindicates that theproposed methodprovides extremely goodapproach for topographic mapping.In future, this research can be continued to identify level, gentle, moderate, steep and very steep terrains. Also, addition texture attribute derived from co-occurrence matrix and modern classifier such as support vector machines could be used to improve the accuracy of terrain classification stage.

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